Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE
Advanced Subsidiary Level
Further Mathematics (8FM0)
Paper 22 Further Pure Mathematics 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>1(i)(a)</td><td colspan="2">Identity is d</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td colspan="2"></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>$(b \circ c)^{-1} = a^{-1}$</td><td>Alt: $(b \circ c)^{-1} = c^{-1} \circ b^{-1} = c \circ f$</td><td>M1</td><td>1.1a</td></tr>
<tr><td></td><td>$= a$</td><td>$= a$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td colspan="2">The set cannot be a subgroup because e.g.
G has order 6 and 4 does not divide 6 so by Lagrange's Theorem,
or the identity is not in the set,
or closure fails as e.g. $a \circ a = d$ is not in the set.</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td colspan="2"></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td colspan="2">Table for $\{b, d, f\}$ is
<table>
<tr><th></th><th>b</th><th>d</th><th>f</th></tr>
<tr><td>b</td><td>f</td><td>b</td><td>d</td></tr>
<tr><td>d</td><td>b</td><td>d</td><td>f</td></tr>
<tr><td>f</td><td>d</td><td>f</td><td>b</td></tr>
</table></td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td colspan="2">This is closed, as no new elements, b and f are inverses and d is the identity, so the subset is a subgroup.</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td>(ii)</td><td colspan="2">$y^3xy^3x^2 = y^2yxy^3x^2 = y^2xy^5y^3x^2$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td colspan="2">$= yyxy^8x^2 = yxy^5y^8x^2 = xy^5y^{13}x^2$</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td colspan="2">$= xy^{18}x^2 = xex^2 = x^3 = e$ *</td><td>A1*</td><td>2.1</td></tr>
<tr><td></td><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td colspan="5">(9 marks)</td></tr>
</table>

Notes:

(i)

(a)

B1: Correct element, d, identified.

(b)

M1: Correct method to find the inverse of $b \circ c$. May find $b \circ c$ first and identify its inverse, or may apply inverse property and find $c \circ f$.

A1: Correct element, a.

(c)

B1: Any correct reason given. See scheme for possibilities.

(d)

M1: Investigates closure, e.g. by drawing the table for $\{b,d,f\}$ or finding the individual products (at least 3) or by considering $\langle b \rangle = \{b, b^2, b^3\} = \{b, f, d\}$ so cyclic.

A1: Correctly shows closure, and refers to inverse and identity, and makes conclusion it is a subgroup.

(ii)

M1: Applies the relation $yx = xy^5$ at least once to change positions of an x and a y. May work from either end.

M1: Completes the process of gathering all y's or x's together by using the relation repeatedly. May reduce elements according to order throughout rather than at the end.

A1*: Uses the orders of the elements to reduce gathered y terms and x terms down to identity to complete the proof.

Question	Scheme	Marks	AOs
2 (a)	$(n \equiv) 5x + 6y \pmod{21}$	**B1**	1.1b
		(1)	
(b)	$x + y = 43$	**B1**	1.1b
	$\Rightarrow n \equiv 5(x+y) + y \equiv 5 \times 43 + y \pmod{21}$	**M1**	3.1a
	$\equiv 5 \times 1 + y \equiv 5 + y \pmod{21}$	**A1**	1.1b
	$n \equiv 0 \pmod{21} \Rightarrow 5 + y \equiv 0 \pmod{21} \Rightarrow y \equiv -5 \pmod{21}$	**M1**	1.1b
	So (as $y \ldots 0$) minimum for y is 16	**A1**	2.3
		(5)	
	Alternative $x + y = 43$	**B1**	1.1b
	$5x + 6y \pmod{21} = 0 \Rightarrow \ldots\ 5x + 6y = 21N$ Solve simultaneous $x + y = 43$ and $5x + 6y = 21N$	**M1**	3.1a
	$y = 21N - 215$	**A1**	1.1b
	$N = 11 \Rightarrow y = 21 \times 11 - 215 = \ldots$	**M1**	1.1b
	minimum for y is 16	**A1**	2.3
		(5)	
			(6 marks)

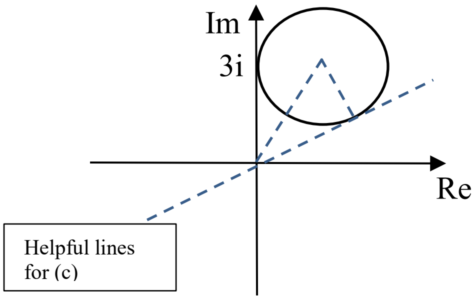
Notes:

(a)
B1: Correct congruence expression.
(b)
B1: Forms the equation in *x* and *y* from the given information.
M1: Substitutes for *x* or *x* + *y* in the congruence expression to form an expression in *y* only. Alternatively, they many eliminate *y* to get an equation in *x* only.
A1: Correct equation in *y* (or *x*) only.
M1: Full method to reach a residue for *y*. If *y* was eliminated from the congruence they must use *x* + *y* = 43 again to form an appropriate expression for *y* here.
A1: Interprets the situation correctly to give 16 for the minimum value for *y*.
Alternative
B1: Forms the equation in *x* and *y* from the given information.
M1: uses their congruence express, sets = 0 and forms an equation in *x*, *y* and a third variable. Solve simultaneous equations to find y in terms of the third variable.
A1: Correct equation for *y*
M1: Uses value for the third variable to find a value for y
A1: Interprets the situation correctly to give 16 for the minimum value for *y*.

Question	Scheme	Marks	AOs
3(a)	Uses entries on the diagonal matrix so eigenvalues are −3 and 8	**B1**	1.2
	$\begin{vmatrix} 3-\lambda & k \\ -5 & 2-\lambda \end{vmatrix} = 0 \Rightarrow (3-\lambda)(2-\lambda)+5k=0$ $\Rightarrow \lambda^2 - 5\lambda + 6 + 5k = 0$ $\Rightarrow 6+5k=-24 \Rightarrow k = \ldots$	**M1**	3.1a
	$k=-6$	**A1**	1.1b
		(3)	
(b)	For $\lambda=-3$ eigenvector equations are $\begin{cases} 6x-"6"y=0 \\ -5x+5y=0 \end{cases} \Rightarrow x, y = \ldots$ OR For $\lambda=8$ eigenvector equations are $\begin{cases} -5x-"6"y=0 \\ -5x-6y=0 \end{cases} \Rightarrow x, y = \ldots$	**M1**	2.1
	(For $\lambda=-3$, $x=y$, for $\lambda=8$, $5x=-6y$ so eigenvectors are) One of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ o.e and $\begin{pmatrix} 6 \\ -5 \end{pmatrix}$ o.e.	**A1**	1.1b
	Both of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ o.e. and $\begin{pmatrix} 6 \\ -5 \end{pmatrix}$ o.e.	**A1**	1.1b
	$\mathbf{P} = \begin{pmatrix} 6 & 1 \\ -5 & 1 \end{pmatrix}$ o.e.	**B1ft**	2.2a
		(4)	
		(7 marks)	

Notes:

(a)
B1: Recalls that the eigenvalues form the entries of the diagonal matrix in the diagonalisation process. May be implied by working or stated, for work in (a) or (b). Implied by an attempt at equating the determinants of the two matrices.
M1: For a full method to find *k*, e.g. by finding the characteristic equation and using (one of) the eigenvalues to find *k*. The factor theorem can be used with either eigenvalue, for example, after finding the characteristic equation.
A1: Correct value.
Note Using $k=-6$
B1: Recalls that the eigenvalues form the entries of the diagonal matrix in the diagonalisation process.
M1: Uses $k=-6$, finds the characteristic equation and solves to find the eigenvalues.
A1: Draws a conclusion, same eigenvalues therefore $k=-6$
(b)
M1: Correct method to find an eigenvector for either of their eigenvalues.
A1: One correct eigenvector – accept any non-zero multiples of their eigenvalues.
A1: A correct eigenvector for each of their eigenvalues.
B1ft: Gives **P** as a matrix with their columns in the correct order for the given diagonal matrix.

Question	Scheme	Marks	AOs
4(a)	 Circle touching the imaginary axis	M1	1.1b
	Correct circle, centre on line $y = 3$	A1	1.1b
		(2)	
(b)	Circle must touch at 3i $\Rightarrow \lvert 3\text{i}-(-3+3\text{i})\rvert = \alpha\lvert 3\text{i}-(1+3\text{i})\rvert \Rightarrow \lvert 3\rvert = \alpha\lvert -1\rvert \Rightarrow \alpha = 3$	B1	2.4
		(1)	
(c)	E.g. Diametrically opposite point to 3i is $x+3\text{i}$ where $\lvert x+3\rvert = 3\lvert x-1\rvert \Rightarrow x+3 = 3x-3 \Rightarrow x = ...$ Or $\lvert x+\text{i}y-(-3+3\text{i})\rvert = 3\lvert x+\text{i}y-(1+3\text{i})\rvert$ $\Rightarrow (x+3)^2+(y-3)^2 = \text{"9"}(x-1)^2+\text{"9"}(y-3)^2$ $\Rightarrow x^2-3x+y^2-6y+9=0 \Rightarrow \left(x-\frac{3}{2}\right)^2+(y-3)^2=\frac{9}{4}$	M1	3.1a
	Centre is $\left(\frac{3}{2}, 3\right)$	A1	1.1b
	Radius is $\frac{3}{2}$	A1	1.1b
	$\beta = \theta-\phi$ where $\theta = \arctan\left(\frac{3}{3/2}\right) = ...$ or $\phi = \arcsin\left(\frac{3/2}{\sqrt{(3/2)^2+3^2}}\right)$	M1	3.1a
	Attempts both $\theta = \arctan\left(\frac{3}{3/2}\right) = ...$ and $\phi = \arcsin\left(\frac{3/2}{\sqrt{(3/2)^2+3^2}}\right)$ $\Rightarrow \beta = \arctan 2 - \arcsin\frac{\sqrt{5}}{5}$ oe method.	M1	2.1
	$=$ awrt 0.644	A1	1.1b
		(6)	
			(9 marks)

Notes:
(a) **M1:** For any circle drawn tangentially to imaginary axis. May be any quadrant. **A1:** Correct circle, in quadrant 1, with centre on line $y = 3$ and tangent to imaginary axis. (b) **B1:** Correct explanation given. (c) **M1:** Correct method to deduce the centre or radius of the circle. May use geometry, or substitute $z = x + \mathrm{i}y$ and find Cartesian equation first etc. **A1:** Correct centre stated or implied. **A1:** Correct radius stated or implied. **M1:** Realises the right angle triangle and use it to find one relevant angle. **M1:** Full method to find the maximum value for β. E.g. Finds both relevant angles and subtracts as shown in scheme, or may use $\frac{\pi}{2} - 2\arcsin\frac{1}{\sqrt{5}}$. **A1:** Correct answer, awrt.

Question	Scheme	Marks	AOs
5(a)	• In the first stage there is one square so $u_1 = 1$ • Each square from u_n to u_{n+1} is replaced by 5 smaller squares, so $u_{n+1} = 5u_n$ • But one of the squares is then removed, so $u_{n+1} = 5u_n - 1$	B1 B1	2.4 3.3
		(2)	
(b)	AE is $\lambda - 5 = 0 \Rightarrow \lambda = 5$	M1	1.1b
	So CF is $w_n = A \times 5^n$	A1	1.1b
	PS try $v_n = k \Rightarrow k = 5k - 1 \Rightarrow k = \ldots \Rightarrow u_n = "A \times 5^n" + "\frac{1}{4}"$	M1	1.1b
	$u_1 = 1 \Rightarrow 1 = A \times 5^1 + \frac{1}{4} \Rightarrow A = \frac{3}{20}$	M1	3.4
	So $u_n = \frac{3}{20} \times 5^n + \frac{1}{4}$ or $u_n = \frac{3}{4} \times 5^{n-1} + \frac{1}{4}$ oe	A1	1.1b
		(5)	
(c)	Each square in stage *n* has area $\frac{25}{9^{n-1}}$ so total area is $\frac{25}{9^7} \times u_8 = \frac{25}{9^7} \times \left(\frac{3}{4} \times 5^7 + \frac{1}{4}\right)$	M1	3.1a
	=0.3062… Accept awrt 0.31	A1	1.1b
		(2)	
		(9 marks)	

Notes:

(a)
B1: For explaining any two of the three aspects in the scheme.
B1: All three aspects explained.

(b)
M1: Sets up and solves the auxiliary equation.
A1: Correct complementary part found.
M1: Selects correct form for particular solution and substitutes and combines result with their CF
M1: Uses the initial value to find the constant.
A1: Correct solution.

(c)
M1: Attempts a scale factor with their u_8. Accept attempts at scaling by 25×3^{-k} or 25×9^{-k} where *k* is 7, 8 or 9.
A1: Correct answer.

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